

# Refitting FVS diameter growth equations across multiple ecoregions

David D. Diaz and Gregory Ettl

Growth Model User Group

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School of Environmental and Forest Sciences

Center for Sustainable Forestry  
at Pack Forest

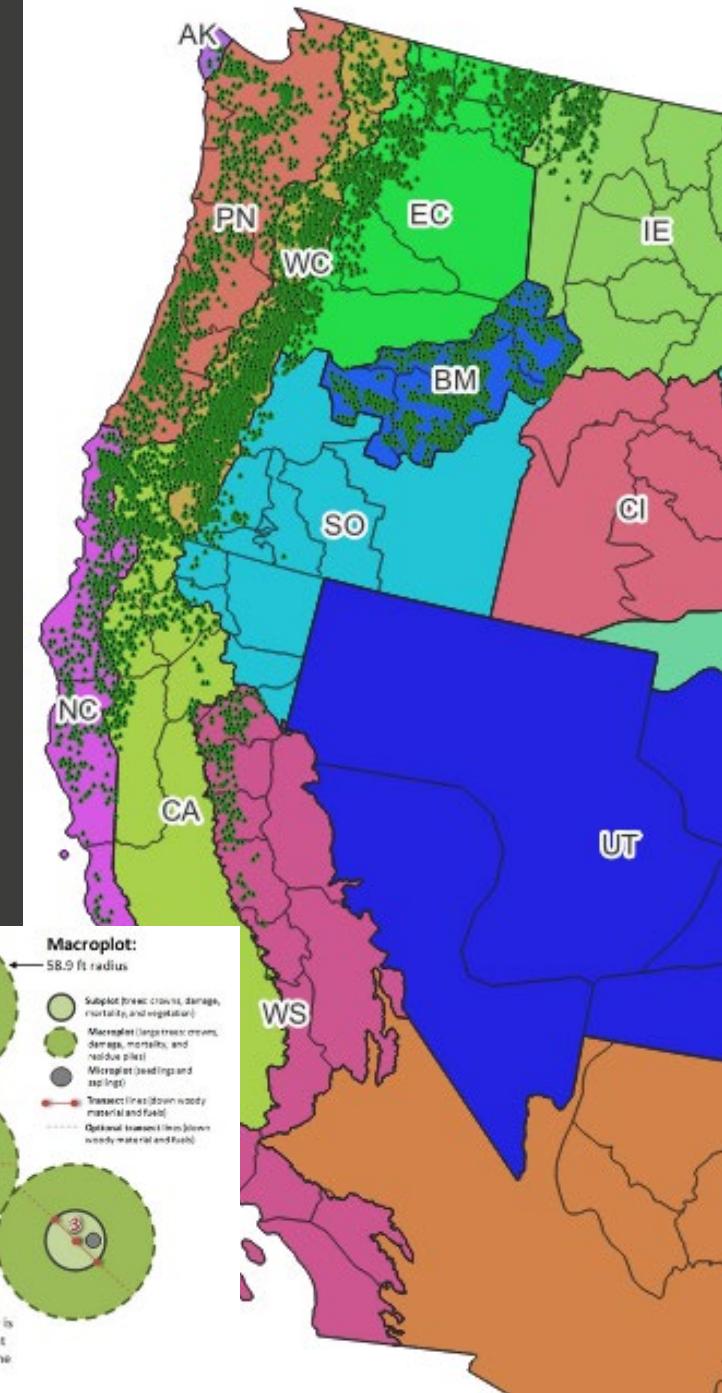
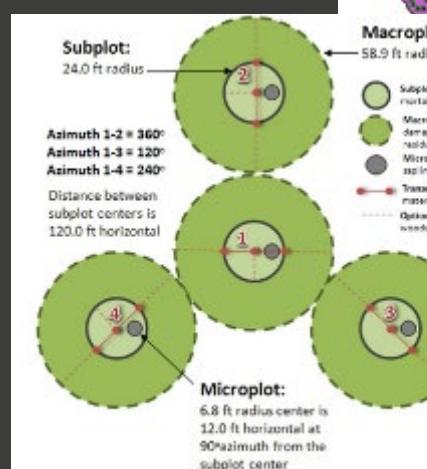
Ecotrust



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# What are we doing?

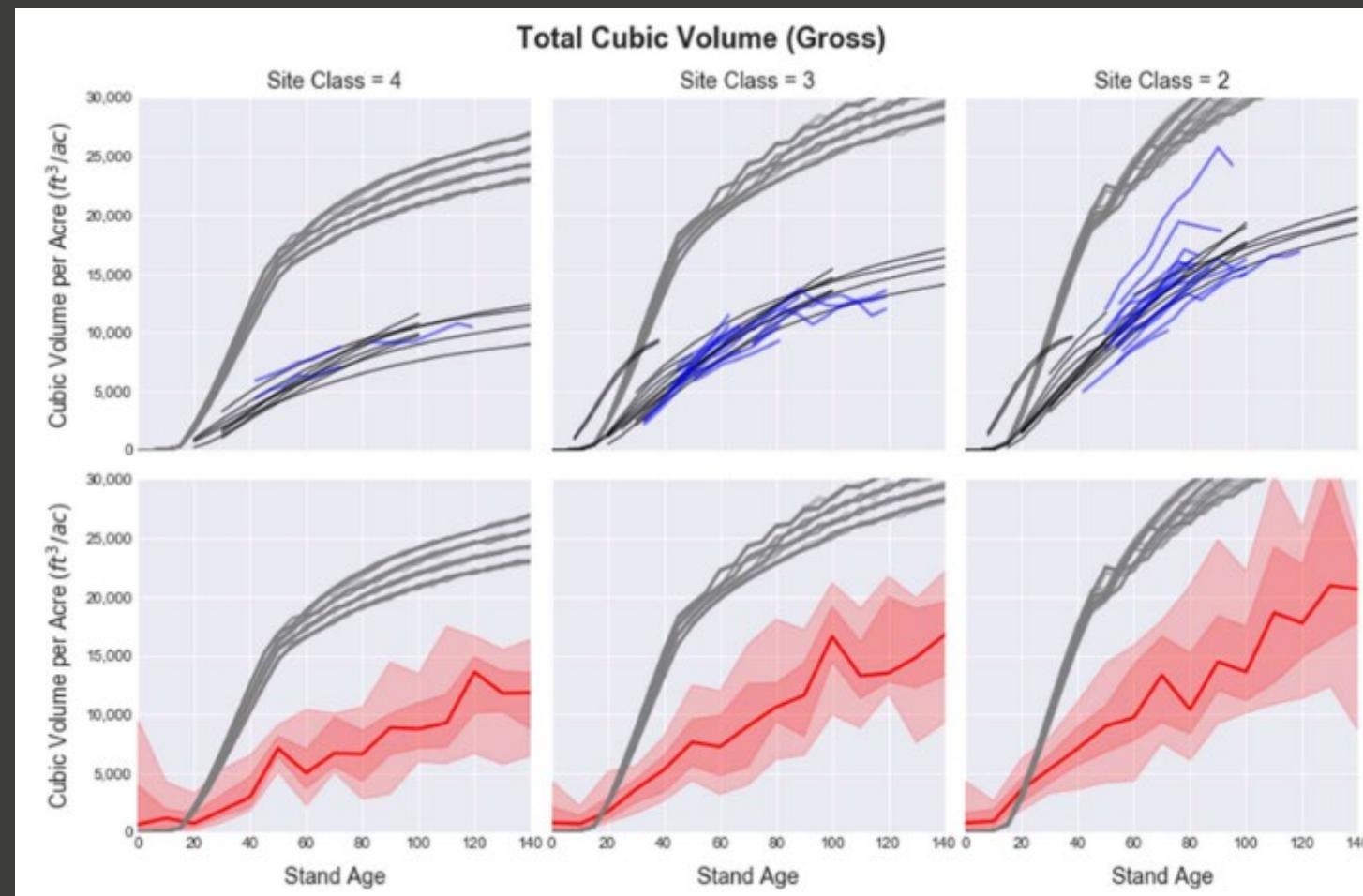
- Refitting Forest Vegetation Simulator (FVS) diameter growth equations from Open Data (FIA) as a recurring and reproducible use case.
- Can we simplify these growth equations and make them more consistent and easily maintained across regions?
- Can we implement a model fitting process that propagates uncertainty while learning simultaneously and borrowing strength across multiple regions?



# Need to Refit FVS

FVS is the most widely-used growth-and-yield model in the USA...

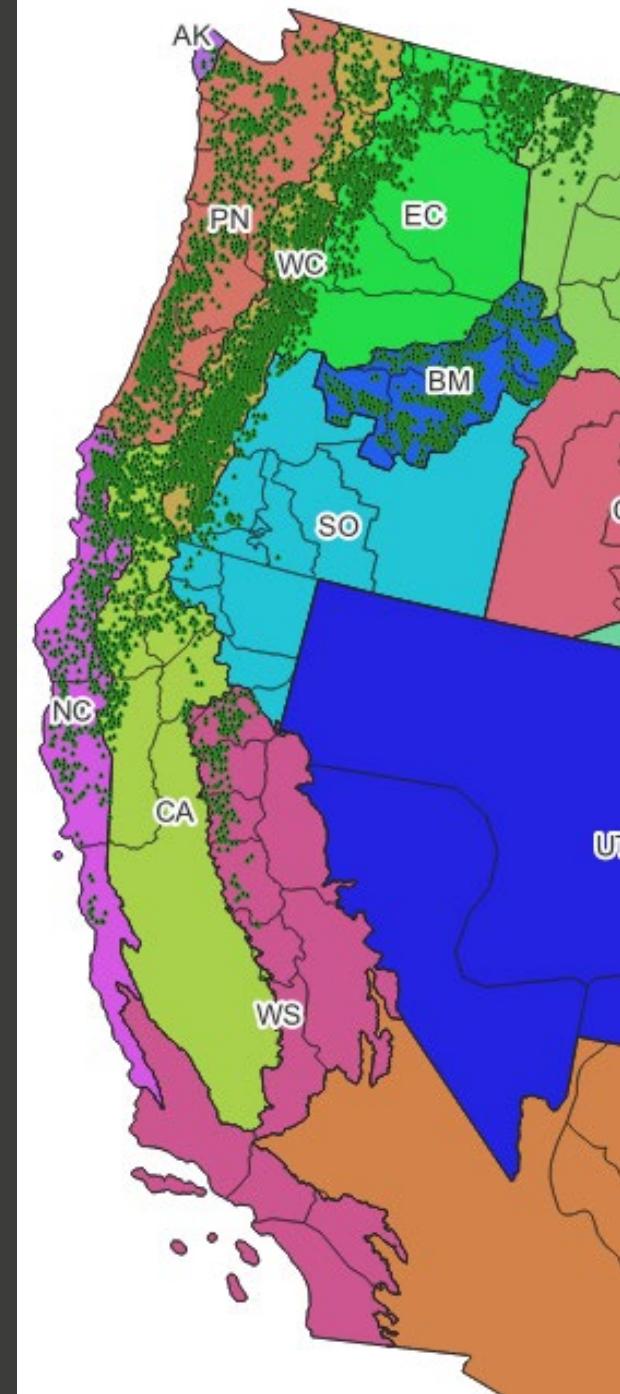
Out-of-the-box, FVS substantially diverges from patterns observed in FIA, long-term permanent plot records, and widely-used yield curves.



FVS PN variant on Douglas-fir. Gray lines show FVS simulations. Blue lines show permanent plot data. Black lines show published yield curves. Red line shows median, and bands show 25th-75th and 10th-90th percentile range of FIA data.

## Filtering FIA Observations

- All trees that have been remeasured on an FIA plot at least once in Oregon, Washington, or California.
- Remove plots with harvest, fire, or geologic disturbance.  
*These are disturbances it is reasonable to expect FVS users to specify.*
- Retain plots where insect, wind, and other disturbances noted.  
*These are disturbances we rarely expect FVS users to (be able to) specify.*
- Remove trees where 10-year DBH change is so negative that it exceeds reasonable measurement error with a diameter tape.

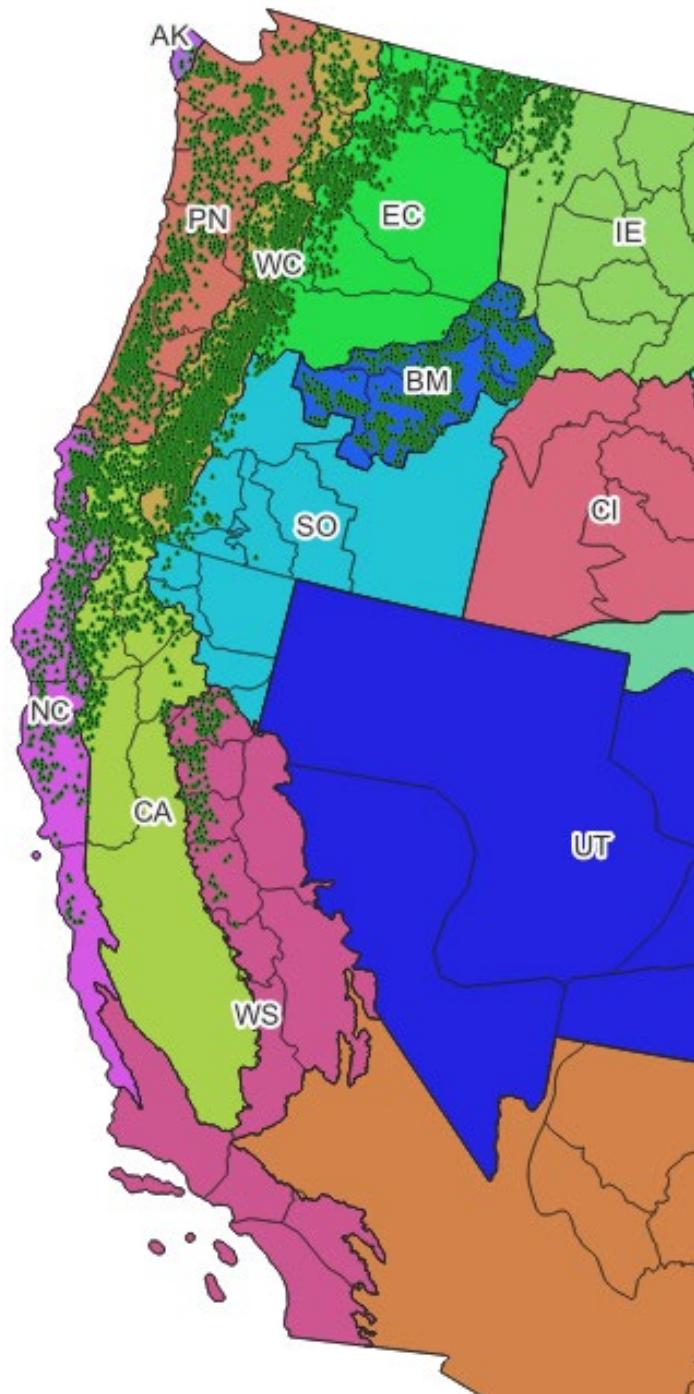


# Repeatedly measured FIA plots

**Table 3.14. Repeatedly measured trees used for model training and evaluation**

SPECIES	FVS REGIONAL VARIANTS									<b>Total</b>
	WC	PN	SO	EC	BM	CA	NC	WS	IE	
Douglas-fir	15,649	10,155	303	5,614	3,318	3,490	3,783	889	803	44,004
Ponderosa pine	293	2	8,315	2,099	6,057	1,023	103	1,078	469	19,439
Western hemlock	6,288	3,571	20	699	--	30	130	--	188	10,926
White fir	891	1	2,706	4	329	1,267	179	2,097	--	7,474
Pacific silver fir	4,114	373	88	932	4	--	--	--	--	5,511
Mountain hemlock	1,836	54	1,022	690	8	83	7	360	9	4,069
Western redcedar	1,567	751	--	338	--	9	4	--	465	3,134
Canyon live oak	8	2	--	--	--	1,470	442	633	--	2,555
Tanoak	5	--	--	--	--	198	1,961	88	--	2,252
Red alder	527	1,189	--	13	5	12	50	--	--	1,796
California black oak	68	4	37	--	--	637	285	576	--	1,607
Pacific madrone	190	65	--	--	--	688	478	11	--	1,432
Coast redwood	--	--	--	--	--	--	1,405	--	--	1,405
Bigleaf maple	424	527	--	30	--	98	100	26	--	1,205
Noble fir	976	20	10	103	1	2	1	--	1	1,114

*Notes: The six most abundant species in each of the four focal ecoregions are shown with tinted cells. Each cell displays the count of unique live trees with repeated observations of DBH available for modeling.*



## THE (abridged) WYKOFF MODEL

**How FVS (often) estimates diameter growth**

$$\text{BAI} = f(\text{SIZE}, \text{ SITE}, \text{ COMP})$$

$$\text{BAI} \sim \text{DBH}_{t+1}^2 - \text{DBH}_t^2$$

$$\text{DDS} = \text{DIB}_{t+1}^2 - \text{DIB}_t^2$$

$$\text{SIZE} = b_0 + b_1 * \ln(\text{DBH}) + b_2 * \text{DBH}^2$$

$$\text{SITE} = b_3 * \ln(\text{SI}) + b_4 * \text{SL} + b_5 * \text{EL} + \varepsilon_{\text{LOC}} + \varepsilon_{\text{PLOT}} \leftarrow \text{added random effects}$$

$$\text{COMP} = b_6 * \text{CR} + b_7 * \text{COMP}_{\text{TREE}} + b_8 * \text{COMP}_{\text{STAND}}$$

$$\text{DDS} = \exp(\text{SIZE} + \text{SITE} + \text{COMP}) + \varepsilon_{\text{TREE}} \leftarrow \text{process error}$$

Where:

BAI is basal area increment; DDS is difference of inside-bark squared diameters; DBH is diameter at breast height; COMP is the combined effects indicating a tree's competitive environment; CR is crown ratio; COMP<sub>STAND</sub> is a stand-level competitive indicator (e.g., crown competition factor); and COMP<sub>TREE</sub> is a tree-level indicator of competitive status (e.g., basal area of larger trees).

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A Basal Area Increment Model for Individual Conifers in the Northern Rocky Mountains

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WILLIAM R. WYKOFF

United States  
Department of  
Agriculture  
Forest Service  
Forest Management  
Service Center  
Fort Collins, CO

Pacific Northwest Coast (PN)  
Variant Overview  
*Forest Vegetation Simulator*

## MAINTAINING THE MODEL

## Model selection

- In each region, FVS employs a variety of covariates, including quadratic terms to predict diameter growth for each tree species.
- In several instances, a species' equation may include collinear covariates, such as multiple indicators of tree-level or stand-level competition.
- Do we need to maintain this many customized equations across species and regions? Can we fit a model using simpler variables that can be calculated directly from inventory data?

$$\begin{aligned}
 \ln(DDS) = & b_1 + (b_2 * EL) + (b_3 * EL^2) + (b_4 * \ln(SI)) + (b_5 * \sin(ASP) * SL) + (b_6 * \cos(ASP) \\
 & * SL) + (b_7 * SL) + (b_8 * SL^2) + (b_9 * \ln(DBH)) + (b_{10} * CR) + (b_{11} * CR^2) + \\
 & (b_{12} * DBH^2) + (b_{13} * BAL / (\ln(DBH + 1.0))) + (b_{14} * PCCR) + (b_{15} * RELHT) + \\
 & (b_{16} * \ln(BA)) + (b_{17} * BAL) + (b_{18} * BA)
 \end{aligned}$$

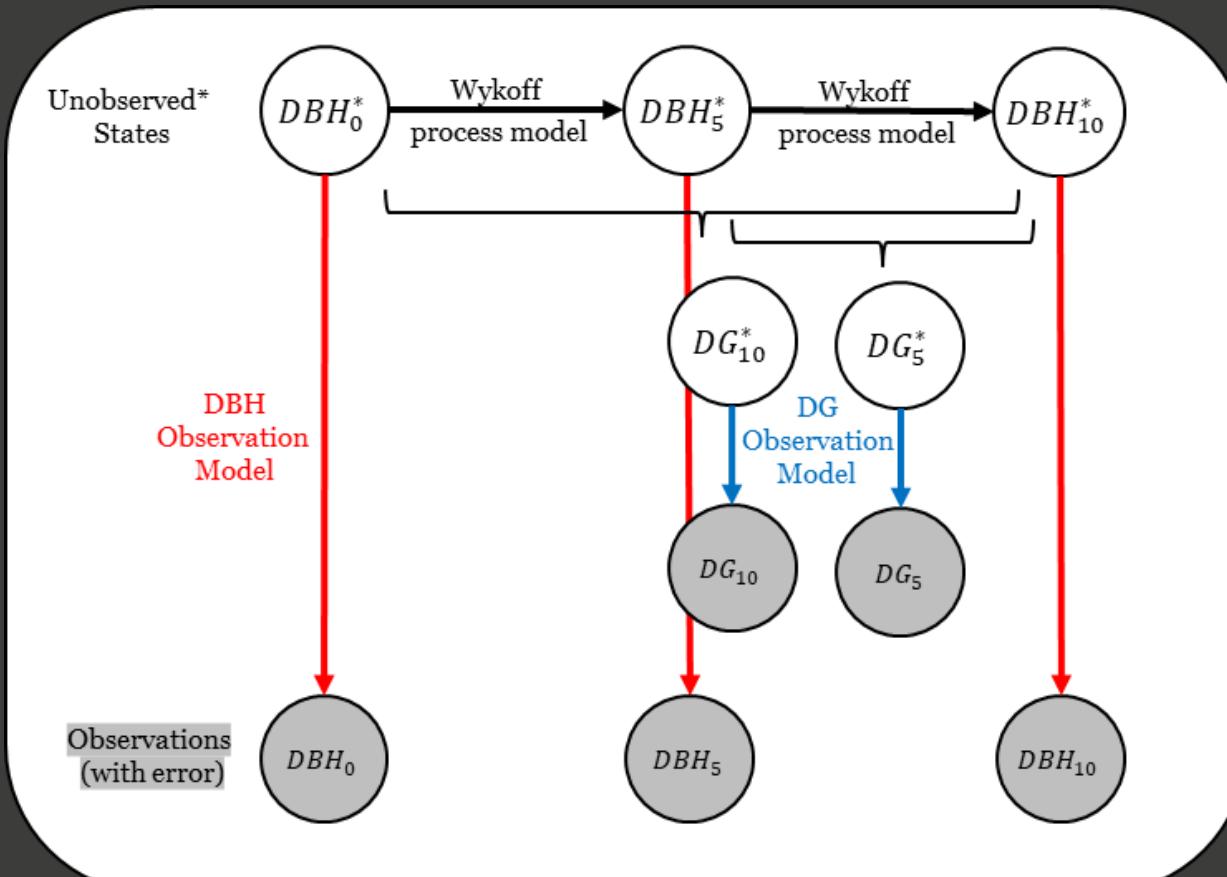
### A Basal Area Increment Model for Individual Conifers in the Northern Rocky Mountains

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# Bayesian State Space Model with Errors-in-Variables



## TREE GROWTH INFERENCE AND PREDICTION FROM DIAMETER CENSUSES AND RING WIDTHS

JAMES S. CLARK,<sup>1,2,3,4,6</sup> MICHAEL WOLOSIN,<sup>2,3</sup> MICHAEL DIETZE,<sup>2,3</sup> INÉS IBÁÑEZ,<sup>2,3</sup> SHANNON LADEAU,<sup>2,3,7</sup> MIRANDA WELSH,<sup>1</sup> AND BRIAN KLOEPPEL<sup>5</sup>

<sup>1</sup>Nicholas School of the Environment, Duke University, Durham, North Carolina 27708 USA

<sup>2</sup>Department of Biology, Duke University, Durham, North Carolina 27708 USA

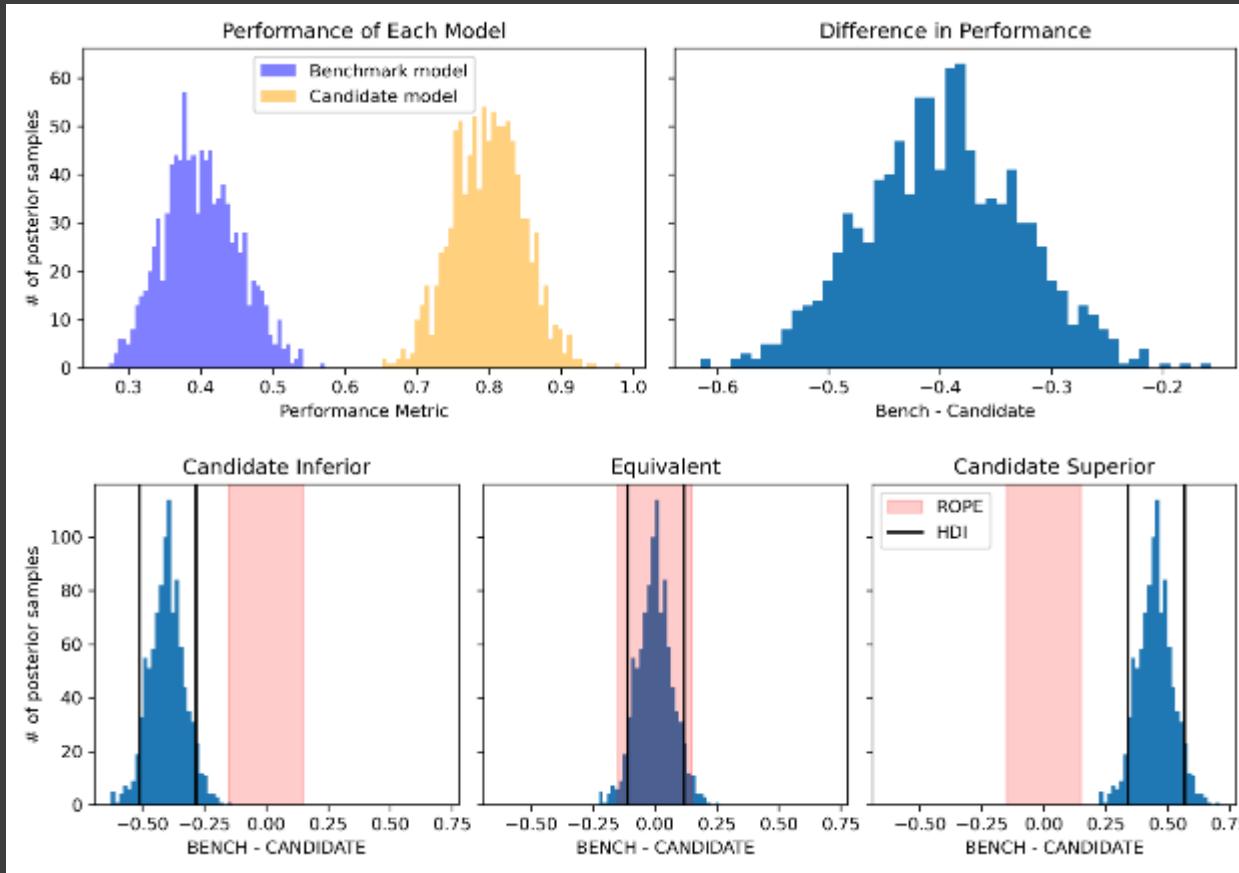
<sup>3</sup>University Program in Ecology, Duke University, Durham, North Carolina 27708 USA

<sup>4</sup>Institute of Statistics and Decision Sciences, Duke University, Durham, North Carolina 27708 USA

<sup>5</sup>Coweeta Hydrologic Laboratory, Otto, North Carolina 28763 USA

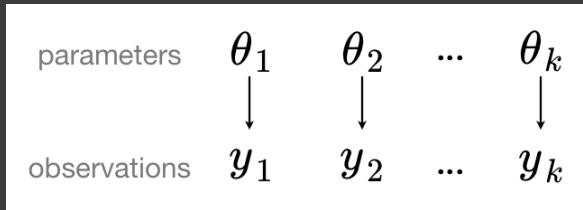
- True diameter for a tree evolves on a periodic time step and is always measured with error. This evolution of true and observed DBHs and increments is described using a Bayesian State Space model.
- Periodic measurements of tree diameter are available at ~10-year intervals.
- Some trees also have increment cores collected that show incremental growth at 5-year intervals.
- The model is implemented in Numpyro and fit using Markov Chain Monte Carlo (NUTS).

## HDI-ROPE decision rule



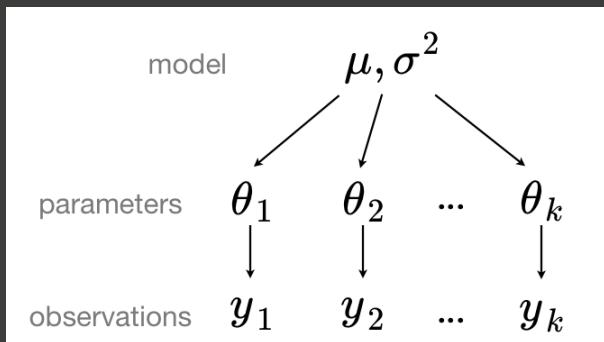
- Using Mean Absolute Error of diameter growth predictions as a performance metric, we compare the distribution of errors for each model against our benchmark/preferred model.
- We define a Region of Practical Equivalence (ROPE) that models should not be judged superior/inferior from one another if their performance differs by <1% of the average 10-year diameter growth for a tree species.
- This leads to four possible outcomes for a candidate model relative to the benchmark: inferior, equivalent, superior, or inconclusive (difference between models overlaps ROPE).

# Learning to borrow strength



## UNPOOLED / INDEPENDENT REGIONS – CURRENT FVS PATTERN

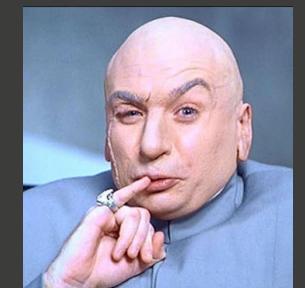
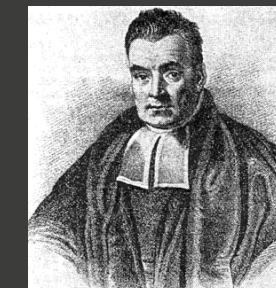
Independent sets of model parameters ( $\theta$ ) are fit in each ecoregion to observations only from that ecoregion



## HIERARCHICAL / PARTIALLY POOLED – PROPOSED PATTERN

Model parameters in each ecoregion are constrained to follow a parent distribution shared across ecoregions.

Image credit: Chris Fonnesbeck  
<https://github.com/widdowquinn/Teaching-Stan-Hierarchical-Modelling/tree/master/images>



Join the Bayesian conspiracy!

EMBRACING SIMPLER MODELS  
with hierarchical forms

- For every species and region, there was not a single candidate model deemed superior to the benchmark model.

(Out of 17 candidates among “full” and “simple” Wykoff models choosing from three tree-level and three stand-level competition alternatives.)

- Unpooled benchmark models were also never deemed superior to hierarchical models for any species or region.

**Table 3.16. Summary of Unpooled Model Comparisons**

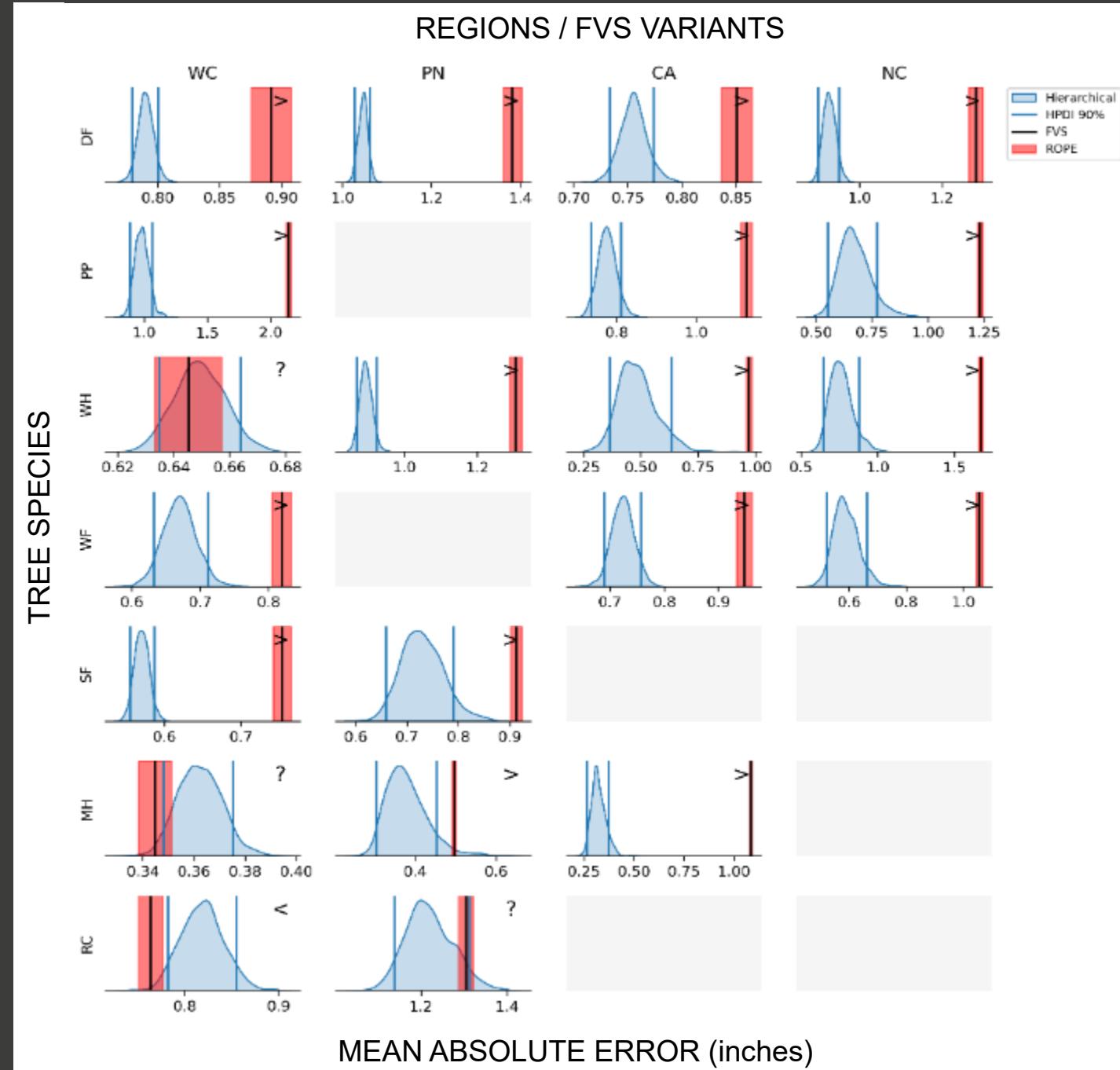
**HDI-ROPE decisions within each FVS Regional Variant**

<b>Common Name</b>	<b>WC</b>				<b>PN</b>				<b>CA</b>				<b>NC</b>			
	?	<	=	>	?	<	=	>	?	<	=	>	?	<	=	>
Douglas-fir	5	7	5	0	9	6	2	0	9	6	2	0	9	6	2	0
Ponderosa pine	17	0	0	0	17	0	0	0	14	3	0	0	17	0	0	0
Western hemlock	11	4	2	0	11	6	0	0	17	0	0	0	17	0	0	0
White fir	17	0	0	0	17	0	0	0	16	1	0	0	17	0	0	0
Pacific silver fir	13	0	4	0	17	0	0	0	--	--	--	--	--	--	--	--
Mountain hemlock	16	0	1	0	17	0	0	0	17	0	0	0	17	0	0	0
Western redcedar	16	1	0	0	17	0	0	0	17	0	0	0	17	0	0	0
Canyon live oak	17	0	0	0	17	0	0	0	17	0	0	0	17	0	0	0
Tanoak	17	0	0	0	--	--	--	--	17	0	0	0	17	0	0	0
Red alder	17	0	0	0	13	4	0	0	17	0	0	0	17	0	0	0
California black oak	17	0	0	0	17	0	0	0	17	0	0	0	17	0	0	0
Pacific madrone	17	0	0	0	17	0	0	0	17	0	0	0	17	0	0	0
Coast redwood	--	--	--	--	--	--	--	--	--	--	--	--	14	3	0	0
Bigleaf maple	17	0	0	0	14	3	0	0	17	0	0	0	17	0	0	0
Noble fir	16	0	1	0	17	0	0	0	17	0	0	0	17	0	0	0

Notes: for each of the four focal regional variant of FVS (WC = Westside Cascades, PN = Pacific Northwest Coast, CA = Inland California and Southern Cascades, and NC = Klamath Mountains), 17 candidate models were compared using the HDI-ROPE decision rule against the SIMPLE-BALLNDBH-BAL benchmark model. The values in each cell correspond to the count of candidate models for a species in each ecoregion categorized as one of four model comparison outcomes: inconclusive (?); inferior than benchmark (<); equivalent to benchmark (=), and superior to benchmark (>). Regions where a species was never observed are indicated with “--”.

DID ANYTHING IMPROVE?  
compared to existing FVS predictions

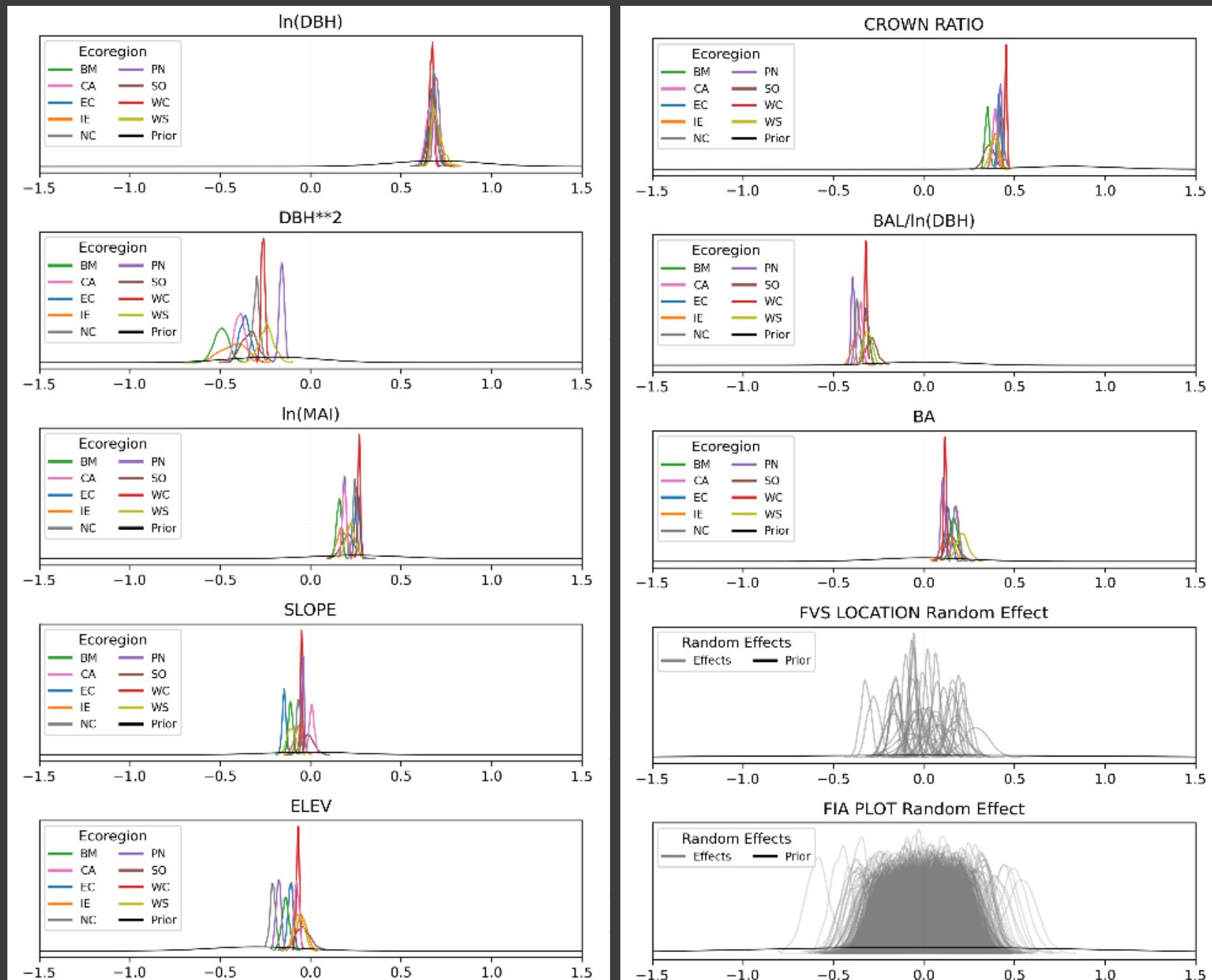
- In 25 out of 43 instances (regions where species is observed) newly-fitted hierarchical benchmark models were superior to FVS as judged by HDI-ROPE using MAE
- FVS outperformed newly-fitted models in 6 out of 43 instances.
- Model comparison inconclusive in 8 out of 43 instances.
- FVS had an overprediction bias for all species in all regions.



# What really matters

- Tree size and crown ratio have strong positive effects.
- Site productivity is positive but less impactful.
- Slope and elevation effects are commonly near zero.
- Tree-level competition indicator always negative. Stand-level competition almost always positive and smaller in magnitude.
- Substantial variation left unexplained at location and plot-levels.

Standardized Coefficients in Douglas-fir Benchmark Model



## GROWTH-AND-YIELD

### Key Findings

- Open Data from FIA provides a large and diverse data stream that can be used to refit FVS on a recurring basis.
- FVS diameter growth equations can be simplified without reducing performance for all species and regions examined.
- A hierarchical Bayesian approach allows for models to be fit simultaneously across multiple regions spanning broad geographic extents, which may be particularly valuable when new climate-aware models are developed.  
Bayesian models also intuitively capture model uncertainty and accommodate multiple sources of observations with error. This may eliminate the need to maintain independent FVS variants.

# Thank you.

[ddiaz@vibrantplanet.net](mailto:ddiaz@vibrantplanet.net)



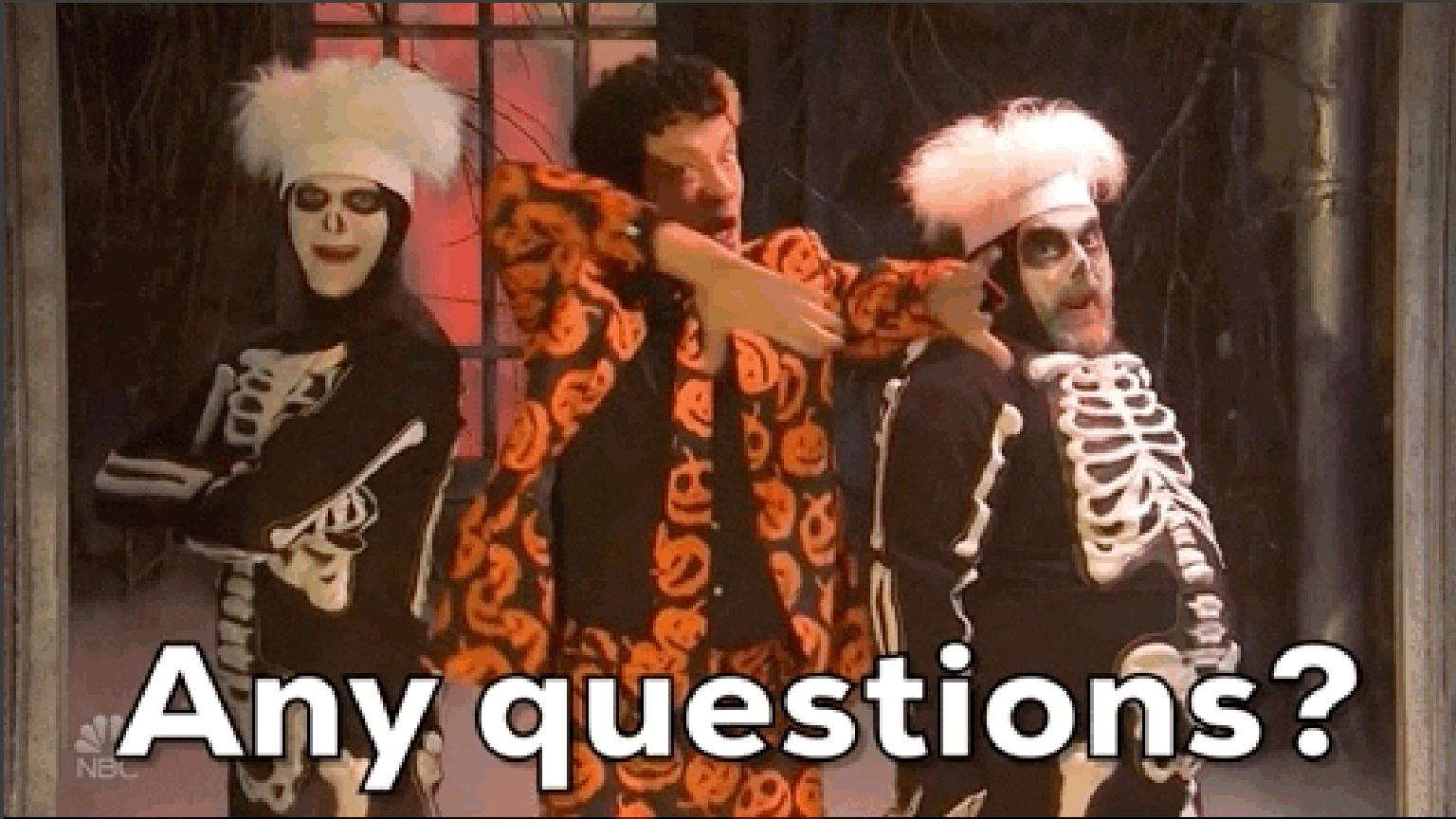
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**Any questions?**